Proximity-induced transport in superconductor - normal metal nanostructures

1998 J. Phys.: Condens. Matter 10 L615

(http://iopscience.iop.org/0953-8984/10/35/003)

J. Phys.: Condens. Matter **10** (1998) L615–L621. Printed in the UK PII: S0953-8984(98)95376-0

LETTER TO THE EDITOR

Proximity-induced transport in superconductor–normal metal nanostructures

R Seviour†, C J Lambert† and A F Volkov‡

† School of Physics and Chemistry, Lancaster University, Lancaster LA1 4YB, UK
‡ Institute of Radioengineering and Electronics of the Russian Academy of Sciences, Mokhovaya Street 11, Moscow 103907, Russia

Received 26 June 1998

Abstract. We calculate the conductance for a diffusive normal wire (N) in contact with a superconductor (S). Using a numerical scattering matrix approach and a quasiclassical Green function technique, we compare the conductance G of the system connected to two normal reservoirs when the superconductor is in its normal state with the conductance G_s in the superconducting state and predict that the difference $\delta G = G_s - G$ may be negative or positive depending upon the S/N interface resistance and the interface resistance at the ends of the N wire. As the temperature T is varied, $\delta G(T)$ may change sign and exhibit two maxima.

Theoretical studies carried out in recent years have revealed that the proximity effect enhances the conductance of a normal metal (N) film in contact with a superconducting metal (S) strip (see for example [1–4] and reviews [5, 6]). The temperature dependence of the conductance δG caused by the proximity effect has a nonmonotonic behaviour, with the zero-bias conductance δG vanishing at $T = 0$, reaching a maximum at $T \approx \epsilon_{Th}$ then tending to zero for $T \gg \epsilon_{Th}$ [1, 3, 4], where $\epsilon_{Th} = D/L^2$ is the Thouless energy, D is the diffusion coefficient. A similar dependence occurs for δG as a function of the voltage, at zero temperature [2]. This nonmonotonic behaviour was first predicted in [7] where a short ScN (c meaning constriction) contact was analysed. Using the assumption $\epsilon_{Th} \gg \Delta$ it was shown that δG reaches a maximum at $T \approx \Delta$. Therefore, one can state that in the general case the conductance variation has a maximum at $T \approx \min[\Delta, \epsilon_{Th}]$. Such nonmonotonic behaviour of δG has been observed experimentally both in short ScN contacts [8] and in mesoscopic S/N structures where $\epsilon_{Th} \ll \Delta$ [9–11].

The above statement about positive δG refers to the conductance variation of the normal wire δG_N only, whereas the experimentally measured conductance variation δG of the whole system includes not only δG_N but also a variation of the interface conductances $\delta G_{S/N}$ and $\delta G_{N/N'}$ (see figure 1) where N′ denotes the normal reservoirs. Evidently the N/N′ interface resistance can lead to a negative variation of the conductance δG of the system as $\delta G_{N/N'}$ is determined by a variation of the density of states (DOS) in the normal film near the N/N′ interface and below T_c the DOS at small energies in the N film is decreased [12, 13]. This decrease of the DOS in the N film was measured in [14]. If $\delta G_{N/N'}$ dominates over δG_N, then δG is negative. An applied magnetic field H or a finite phase difference ϕ (where $|\phi| < \pi/2$) between two superconductors attached to the N film leads to a suppression of the proximity effect and an increase of the system conductance. A transition from a decrease of δG with increasing H or ϕ to an increase of δG has been predicted in [15] and [16] for the zero-temperature case. For finite temperatures this effect was studied recently in [17] and [18]. Another reason for a negative δG is the shunting effect of the S strip. In the superconducting state the S/N interface resistance is increased and therefore the shunting effect of the S strip is weakened. In this letter we examine an N conductor of length $2L$ in the presence of a superconducting contact of length $2L_1$, and show that if the ratio of the S/N interface conductance to the conductance of the N film is less than $(L_1/L)^2$ then the variation of the total conductance δG is negative even in the absence of the N/N′ interface resistance. In this case an applied magnetic field enhances the effect of negative δG. We will present the temperature dependence of $\delta G(T)$ for different $(R_{S/N}/R_L) \equiv g_{b1}^{-1}$ and $(R_{N/N'}/R_L) \equiv r_{b2}$ and show that the sign of the overall conductance variation depends on g_{b1} and r_{b2} in a complicated way. It is interesting to note that $\delta G(T)$ has a peak not only at $T_1 \approx \epsilon_{Th}$ but also at a temperature T_2 close to T_c when $\Delta(T_2) \approx \epsilon_{Th}$.

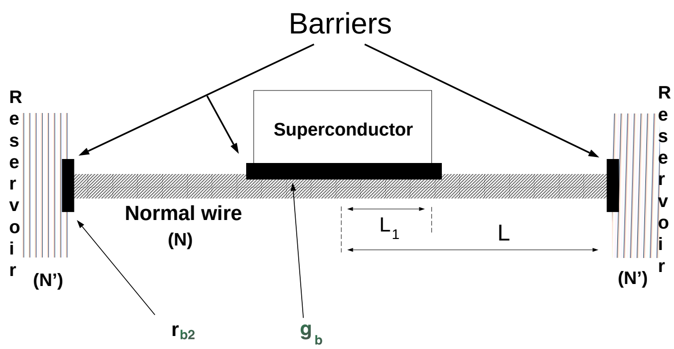

Figure 1. The structure considered.

We consider the system shown in figure 1. The distribution functions in the reservoirs N′ are assumed to be at equilibrium and the condensate functions $F^{R(A)}$ are equal to zero. We assume also that the S/N interface resistance $R_{S/N}$ is larger than the resistance of the N film. In this case the proximity effect is weak; this means that the amplitudes of the condensate functions $F^{R(A)}$ induced in the N film are small and therefore all the variations of the interface conductances ($\delta G_{S/N}$, $\delta G_{N,N'}$) and the N film conductance (δG_N) are small compared with the conductance of the system in the normal state [2, 3, 15]. We consider an arbitary N/N′ interface resistance and restrict the analysis to the diffusive regime where the mean free path is shorter than the coherence length. To calculate the conductance variation δG, we first employ the quasiclassical Green function technique [1, 2, 4, 6, 7, 15] which is valid when quantum corrections to the conductance are not important ($G \gg e^2/h$) and also use a scattering matrix approach which allows one to study the case of arbitrary G [2, 5, 6]. The distribution function f obeys the equation [3, 15, 19]

$$L^2\partial_x\big((1-m)\partial_x f\big) = f g_b G_b \vartheta(x \in (\mathrm{S/N})) \tag{1}$$

where the function $\vartheta(x)$ is equal to 1 in the S/N region and zero otherwise, $m = \mathrm{Tr}(\hat{F}^R - \hat{F}^A)^2/8$ and $\hat{F}^{R(A)}$ is the retarded (advanced) Green function, $g_b = \rho L^2/R_{b\square}d = g_{b1}(L/L_1)$, g_{b1} is the ratio of the normal film resistance to the S/N resistance, $R_{b\square}$ is the S/N interface resistance per unit area in the normal state, ρ and d are the specific resistivity and the thickness of the normal film. The function $G_b(x)$ determines the local normalized conductance of the S/N interface in the superconducting state,

$$G_b(x) = \nu_s\nu_n + \mathrm{Tr}(\hat{F}^R + \hat{F}^A)(\hat{F}_s^R + \hat{F}_s^A)/8 \tag{2}$$

where the DOS in the superconductor $\nu_S = \mathrm{Re}(\epsilon + \mathrm{i}\Gamma)/\sqrt{(\epsilon + \mathrm{i}\Gamma)^2 - \Delta^2}$ and ν_N is the DOS in the normal film (for simplicity we assume $\nu_n = 1$ so that $\nu_S = \nu_N$ when $\Delta = 0$). The first term in equation (2) describes the contribution of the quasiparticle current to the conductance (if $\Gamma = 0$ it differs from zero only at energies $|\epsilon| > \Delta$). The second term is due to Andreev reflection and describes a conversion of the low-energy quasiparticle current into the condensate current (if $\Gamma = 0$, the current is not zero for $|\epsilon| < \Delta$). The condensate functions $\hat{F}_s^{R(A)}$ in the superconductor are assumed undisturbed by the proximity effect, and they are equal to $\hat{F}_s^{R(A)} = \mathrm{i}\hat{\tau}_y F_s^{R(A)}$, where $F_s^{R(A)} = \Delta/\sqrt{(\epsilon + \mathrm{i}\Gamma)^2 - \Delta^2}$, Γ is the damping rate in the excitation spectrum of the superconductor. Equation (1) is supplemented by the boundary condition at $x = L$ [20–22],

$$Lr_{b2}(1 - m)\delta_x f = \nu\big[F_v - f\big] \tag{3}$$

where $r_{b2} = R_{N/N'}/R_L$, $R_{N/N'}$ and R_L are the resistances of the N/N′ interface and of the normal film respectively; $\nu(\epsilon) = (G^R - G^A)/2$ is the DOS of the normal film at $x = L$. $F_v = [\tanh(\epsilon + eV)\,\beta - \tanh(\epsilon - eV)\,\beta]/2$ is the difference of the distribution functions in the N′ reservoirs. The retarded (advanced) Green functions $F^{R(A)}$ obey the linearized Usadel equation. The details of calculations are similar to those of [2, 3, 15, 19], and yield the following expression for the normalized variation of the conductance,

$$\delta S = \frac{(G_s - G_n)}{G_n} = \delta S_1 + \delta S_2 + \delta S_3. \tag{4}$$

Here δS_i ($i = 1,\ 2,\ 3$) are the normalized conductance variations of the conductances of the S/N, N/N′ interfaces and of the N film respectively and can be obtained 'partial' normalized conductances, via the relations

$$\delta S_i = \int_0^\infty \partial\epsilon\beta F_v'(\epsilon)\delta S_{ip}.(\epsilon). \tag{5}$$

Here $\delta S_{1p}(\epsilon) = -\langle m\rangle/(1 + r_{b2})$, $\langle m\rangle = \mathrm{Tr}\langle(\hat{F}^R - \hat{F}^A)^2\rangle/8$; $\delta S_{2p}(\epsilon) = r_{b2}\delta\nu/(1 + r_{b2})$, $\delta\nu = \mathrm{Re}(F^R)^2/2$ at $x = L$, $\delta S_{3p}(\epsilon) = [l_1(m_{b1} - \langle m_{b1}\rangle) - g_b l_1^3/3]/(1 + r_{b2})$. The angle brackets mean the spatial averaging over the region $(0, L_1)$ and $(0, L)$; $m_b(x) = (g_b/L^2)\int_0^x \mathrm{d}y y G_b(y)$, $F_v' = \beta^{-1}\partial_\epsilon F_v$. Finding the retarded (advanced) condensate Green functions $F^{R(A)}(\epsilon, x)$ from the linearized Usadel equation, we can calculate all the quantities in equation (4) determining δS.

In figure 2 we show the dependence of $\delta\tilde{S} \equiv \delta S(1 + r_{b2})/(g_b l_1)^2$ on the inverse temperature $\alpha = \epsilon_{Th}/(2T)$ for different normalized interface conductances r_{b2}^{-1} and g_b. The normalized width of the superconducting strip $l_1 = L_1/L$ is equal to 0.2. One can see that in the absence of the N/N′ resistance ($r_{b2} = 0$) for $g_b = 1.0$, the variation of the conductance δS is negative over a wide temperature range, increasing as T increases with a flat maximum at $T \approx \epsilon_{Th}$ ($\alpha \leqslant 1$). By increasing r_{b2} the conductance variation becomes positive and the peak at the Thouless energy increases significantly. This change in the $\delta S(T)$ dependence is caused by an increase of the N film conductance δS_3 due to an enhancement of the proximity effect with increasing r_{b2}.

In the case of larger S/N interface conductance ($g_b = 2$) the conductance variation δS is positive for relatively low temperatures (figure 2(b)) with a flat maximum at $T \approx \epsilon_{Th}$. This maximum again becomes more pronounced with increasing r_{b2}. We see that in both cases δS is positive near T_c where it has a second maximum, occuring at a temperature T_2 where $\Delta(T_2)$ is comparable to the the Thouless energy. To our knowledge this maximum has neither been observed nor discussed in the literature. We will analyse the nature of this maximum and conditions under which it can be observed elsewhere.

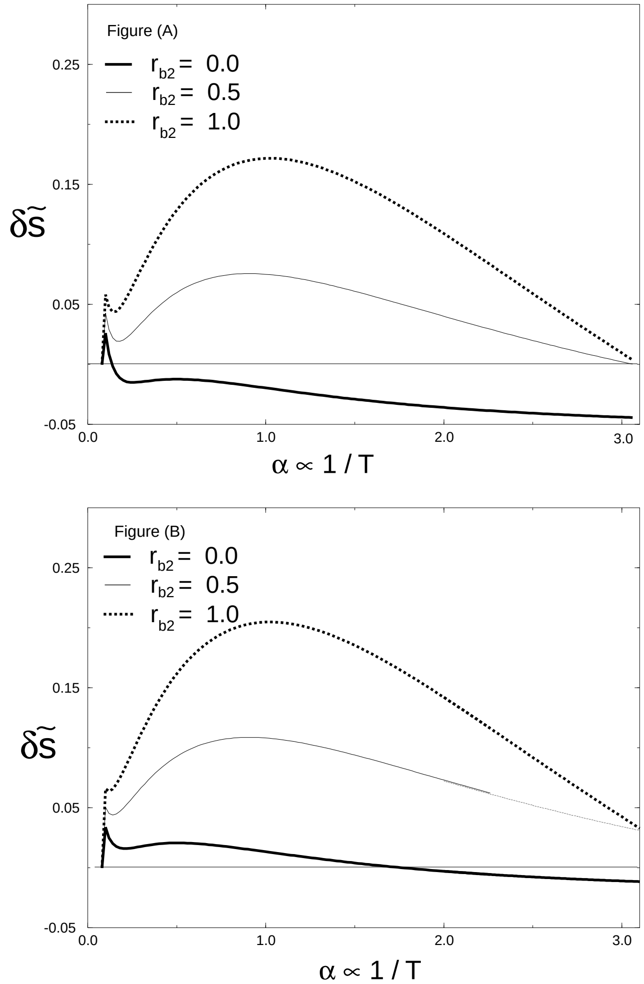

Figure 2. The variation of δS with α (inverse temperature), with the parameters $l_1 = 0.2, \gamma = 0.1$ and $\Gamma = 0.1$ at zero voltage. Figure (A) shows δS for $r_{b2} = 0, r_{b2} = 0.5$ and $r_{b2} = 1$ with $g_b = 1$, figure (B) shows δS for $r_{b2} = 0, r_{b2} = 0.5$ and $r_{b2} = 1$ with $g_b = 2$. The depairing rate in the N film (γ) and in the superconductor (Γ) are measured in units ϵ_{Th}; we also set the zero temperature $\Delta(0) = 10\ \epsilon_{Th}$.

In figure 3 $\delta S(T)$ is shown for the case of a narrow superconducting strip ($l_1 = 0.03$). In this case δS is positive over a wide temperature range ($\alpha < 4$). With increasing r_{b2} the value of T at which δS changes sign increases, and δS becomes negative for example at $\alpha > 1.5$ ($r_{b2} = 5$) in accordance with the results of [15] and [16]. With increasing T the conductance variation reaches a maximum at $T \approx \epsilon_{Th}$.

It is important to note that the changing sign of δS when the parameters of the system (T, r_{b2}, g_b) are varying implies a transition of the phase dependence of δS_3 in the case of a system with two superconductors. If one considers two S strips attached to the N film symmetrically (or a cross geometry investigated in [23]), then the dependence of the conductance of the system on the phase difference ϕ is determined by the simple formula $G = G_n + \delta G(1 + \cos(\phi))$. Thus the conductance G decreases (increases) with increasing

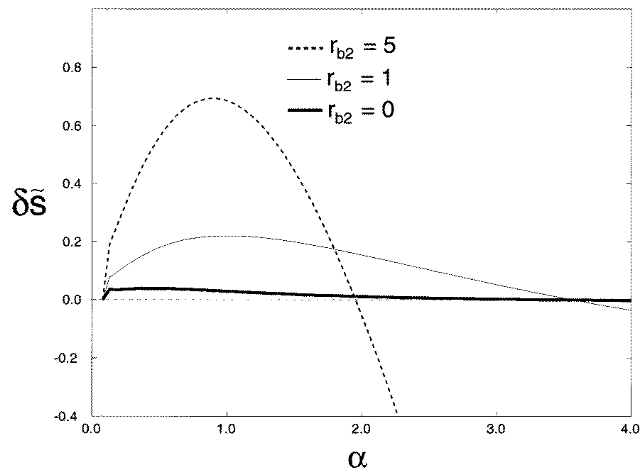

Figure 3. The variation of δS with α (inverse temperature), with the parameters $l_1 = 0.03$, $\gamma = 0.1$, $\Gamma = 0.1$, $r_{b2} = 0$, $r_{b2} = 1$ and $r_{b2} = 5$ with $g_b = 1$ at zero voltage.

ϕ in the case of positive (negative) $\delta G = \delta S G_n$ (at $|\phi| < \pi$).

As an independent check of the above predictions we have used the scattering approach reviewed in [2] to determine δG, for a tight binding lattice with the geometry of figure 1. The numerical approach allows us to investigate the regime that lies outside the validity of the quasiclassical approach used above, namely at zero temperature and incorporating fully quantum corrections to the conductance.

In the linear-response limit, the conductance of a phase-coherent structure may be calculated from the fundamental current–voltage relationship [24], which expresses the conductance in terms of Andreev ($T_a(R_a)$) and normal ($T_0(R_0)$) transmission (reflection) coeffcients. As noted in [26], in the presence of disorder, the various transmission and reflection coeffcients can be computed by solving the Bogoliubov–de Gennes equation. By numerically solving for the scattering matrix, exact results for the dc conductance can be obtained [16, 24, 25].

For the structure shown in figure 1, with a superconductor of length $2L_1$ lattice sites, and an effective barrier resistance $R_{S/N}$ between the superconductor and the normal metal, we have calculated the ensemble averaged conductance variation $\langle\delta G\rangle$ ($\langle\delta G\rangle = \langle G - G_s\rangle$) in units of $2e^2/h$, for a normal diffusive region of dimensions 40 sites wide and 64 sites long, with no barrier between the normal wire and the reservior (i.e. $r_{b2} = 0$). The superconductor is of width 20 sites with an order parameter of magnitude $\Delta_0 = 0.1$ ($\Delta_0 = 0$) in the superconducting (normal) state. Results were obtained by averaging over 100 disorder realizations, yielding an estimated error in the mean values of approximately 0.04.

At zero temperature and bias, for $L_1 = 15$ and $R_{S/N} = 2$, $\langle\delta G\rangle = -0.30$; reducing $R_{S/N}$ to 0.5 yields $\langle\delta G\rangle = 0.16$ confirming the above predictions of quasiclassical theory about the change of sign of $\langle\delta G\rangle$. At finite temperature ($k_B T = \epsilon_{Th}$) for $L_1 = 15$ and $R_{S/N} = 2$, $\langle\delta G\rangle = -0.12$, again confirming the above results.

Using quasiclassical theory we have established that the conductance variation δG of the system shown in figure 1 due to the proximity effect may be either positive or negative depending on the parameters of the system (the interface resistances, temperature etc). For selected parameter values this result was confirmed using a numerical scattering approach. If the S/N interface resistance is large enough ($g_b l_1 < l_1$) and the N/N′ interface resistance is small, then δG is negative over a wide temperature range. If the N/N′ interface resistance is not small compared to the resistance of the N film, then δG is negative at low temperatures. In the case of negative δG, the conductance of a system with two superconductors will increase with increasing phase difference (at $\phi < \pi$). In [23] Petrashov *et al* measured the conductance of a mesoscopic S/N cross structure, and observed a negative variation of δG in the Ag/Al structure and a positive δG in the Sb/Al structure. The cross geometry differs greatly from that analysed above, because in this case the shunting effect of the superconductor is absent (the current does not flow across the S/N interface) and the N/N′ interface resisitance seems to be small. Perhaps the negative variation of δG is caused by a nonuniform current distribution, as suggested by Wilhelm *et al* [27]. However there are experiments on S/N structures in which a negative δG has been observed, with a uniform current density distribution [28–30]. The mechanism for the conductance decrease below T_c in these structures maybe related to a contribution of the S/N or the N/N′ interface resistances studied in the present work, although it is difficult to compare our results with experimental data due to the lack of information about the interface resistances. The results obtained by us may stimulate further experiments on structures with a geometry similar to that studied here, with known interface resistances. There are two peaks in the temperature dependence $\delta G(T)$; one of them located at a temperature T_1 in the vicinity of the Thouless energy ϵ_{Th} and another peak corresponding to a temperature T_2 at which $\Delta(T_2) \approx \epsilon_{Th}$. The first maximum in the $\delta G(T)$ dependence (at $\alpha \approx 1$) is well known [1, 3, 4], whereas the second maximum at higher temperatures (small α) is predicted here for the first time. We note a peculiarity (a minimum) in the temperature dependence of the local conductivity was obtained in [27] for the case of a two-dimensional geometry. The nature of this feature is different from that leading to the maximum in $\delta G(T)$ of the two-terminal, one-dimensional geometry analysed above.

We are grateful to the Royal Society, the Russian Fund for Fundamental Research (project 96-02-16663 a) and to CRDF (project RP1-165) for financial support. We also wish to thank the EPSRC for their financial support.

References

[1] Nazarov Yu V and Stoof T H 1996 *Phys. Rev. Lett.* **76** 823
[2] Volkov A F, Allsopp N K and Lambert C J 1996 *J. Phys.: Condens. Matter* **8** 45
[3] Volkov A F and Pavlovskii V V 1996 *Correlated Fermions and Transport in Mesoscopic Systems* ed T Martin, G Montambaux and J Tran Thanh Van (Gif-sur-Yvette: Frontiers) 267–76
[4] Golubov A A, Wilhelm F and Zaikin A D 1996 *Phys. Rev.* B **55** 1123
[5] Beenakker C W J 1997 *Rev. Mod. Phys.* **69** 731
[6] Lambert C J and Raimondi R 1998 *J. Phys.: Condens. Matter* **10** 901
[7] Artemenko S N, Volkov A F and Zaitsev A Z 1979 *Solid State Commun.* **30** 771
[8] Gubankov V N and Margolin N M 1979 *JETP Lett.* **29** 673
[9] Courtois H, Grandit Ph, Mailly D and Pannetier B 1996 *Phys. Rev. Lett.* **76** 130
Charlat D, Courtais H, Grandit Ph, Mailly D, Volkov A F and Pannetier B 1996 *Phys. Rev. Lett.* **77** 4950
[10] den Hartog S G, van Wees B J, Klapwijk T M, Nazarov Yu V and Borghs G 1997 *Phys. Rev.* B **56** 13 738
[11] Petrashov V T, Shaikadarov R Sh, Delsing P and Claeson T 1998 *Pis. Eskp. Teor. Fiz.* **67** 489
[12] Volkov A F, Zaitsev A V and Klapwijk T M 1993 *Physica* C **210** 21
[13] Hekking F W and Nazarov Yu V 1993 *Phys. Rev. Lett.* **71** 1525
Hekking F W and Nazarov Yu V 1994 *Phys. Rev.* B **49** 6847
[14] Esteve D, Pothier H, Gueron S, Birge N O and Devuret M 1996 *Correlated Fermions and Transport in Mesoscopic Systems* ed T Martin, G Montambaux and J Tran Thanh Van (Gif-sur-Yvette: Frontiers) 297–311
[15] Volkov A F and Zaitsev A V1996 *Phys. Rev.* B **53** 9267

[16] Claughton N R, Lambert C J and Raimondi R 1996 *Phys. Rev.* B **53** 9310

[17] Shaikadarov R Sh 1998 *PhD Thesis* Chernogolovka
Petrashov V T, Shaikadarov R Sh *et al* *JETP Lett.* submitted

[18] Antonov V, Takayanagi H, Wilhelm F and Zaikin A D 1998 *Preprint* cond-mat/9803339

[19] Volkov A F and Takayanagi H 1996 *Phys. Rev. Lett.* **76** 4026
Volkov A F and Takayanagi H 1997 *Phys. Rev.* **56** 11 184

[20] Zaitsev A V 1984 *Zh. Eksp. Teor. Fiz.* **86** 1742 (Engl. transl. 1984 *Sov. Phys.–JETP* **59** 1015)

[21] Kupriyanov M Yu and Lukichev V F 1988 *Zh. Eksp. Teor. Fiz.* **94** 139 (Engl. transl. *Sov. Phys.–JETP* **67** 1163)

[22] Lambert C J, Raimondi R, Sweeney V and Volkov A F 1997 *Phys. Rev.* B **55** 6015

[23] Petrashov V T, Antonov V N, Delsing P and Claeson T 1993 *Phys. Rev. Lett.* **70** 347
Petrashov V T, Antonov V N, Delsing P and Claeson T 1995 *Phys. Rev. Lett.* **74** 5268

[24] Lambert C J 1991 *J. Phys.: Condens. Matter* **3** 6579

[25] Lambert C J, Hui V C and Robinson S J 1993 *J. Phys.: Condens. Matter* **5** 4187

[26] Allsopp N K, Hui V C, Lambert C J and Robinson S J 1994 *J. Phys.: Condens. Matter* **6** 10 475

[27] Wilhelm F, Zaikin A D and Courtois H 1998 *Phys. Rev. Lett.* **80** 4289

[28] Santhanam P, Chi C C, Wind S J, Brady M J, Bucchignano J J 1991 *Phys. Rev. Lett.* **66** 2254

[29] Vloeberghs H, Moshchalkov V V, van Haessendonck, Jonckheere R and Bruynseraede Y 1992 *Phys. Rev. Lett.* **69** 1268

[30] Petrashov V T, Antonov V N, Maximov S V and Shaikhaidaroi R Sh 1993 *JETP Lett.* **58** 49